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DYNAMIC ANALYSIS OF VIGILANTE TOWED CARRIAGE SUSPENSION SYSTEM

TECHNICAL REPORT WAL TR 850/14

BY

KENNETH D. ROBERTSON

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LIGHT AA WEAPON SYSTEM
D/A PROJECT 5010H067

WATERTOWN ARSENAL
WATERTOWN 72, MASS.

Mechanics, vibration

Dynamics

Vigilante suspension system

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TITLE

DYNAMIC ANALYSIS OF VIGILANTE TOWED CARRIAGE
SUSPENSION SYSTEM

ABSTRACT

The suspension system of the Vigilante X-2 towed gun carriage was analyzed theoretically and tested experimentally. A prediction equation for the maximum acceleration transmitted through the suspension system to the carriage proper is developed and the coefficients and exponents of the prediction equation are evaluated by use of a quarter-scale model.

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CONTENTS

NOMENCLATURE

M	= Mass
L_a	= Length of wheel arm
L_c	= Length of carriage between front wheel and rear wheel arm
L_o	= Distance of accelerometer from front wheel. See Figure A-1
$L_{1,2,3...i}$	= Distances of masses 1,2,3...i from front wheel. See Figure A-1
W	= Weight of carriage
I	= Inertia of carriage about front wheel
g	= Acceleration of gravity
S_o	= Relative velocity of carriage with respect to ground
λ	= Wave length of ground profile
A_o	= Wave amplitude of ground profile
K	= Spring constant
t	= Time
m	= Subscript denoting model quantities
p	= Subscript denoting prototype quantities
β	= Angle between carriage frame and horizontal plane. See Figure A-1
η	= Angle between wheel arm and horizontal plane. See Figure A-1
ψ	= Angle between tangent to ground profile and horizontal plane. See Figure A-1
$C_{a,b,c,etc.}$	= Exponents of prediction equation
C_α	= Coefficient of prediction equation
x_1	= Horizontal displacement of point 1, Figure A-1
y_1	= Vertical displacement of point 1, Figure A-1
x_3	= Horizontal displacement of point 3, Figure A-1
v	= Film speed
F_n	= Normal force on wheels
L_f	= Wave length of trace on film
T_i	= Torque about point i

OBJECT

The object of this investigation was to determine the acceleration transmitted to the Vigilante X-2 towed gun carriage through the suspension system when the wheels of the carriage were subjected to a forced displacement similar to that encountered in cross-country travel. A secondary objective was to study the role of the rear suspension system in limiting the acceleration transmitted.

BACKGROUND INFORMATION

The Vigilante X-2 towed gun carriage, subsequently designated as Vigilante gun carriage or simply gun carriage, and its associated equipment have been designed to withstand dynamic loads up to four times the acceleration of gravity. Any dynamic loads in excess of this amount may cause damage to the gun carriage or its associated equipment.

During cross-country travel the gun carriage will be subjected to dynamic loads due to the forced displacement of the wheels caused by the uneven ground profile. The wheels will experience a displacement equal to the amplitude of the ground profile. The resultant displacement and acceleration of the gun carriage will be determined by the suspension system.

The Vigilante gun carriage is designed to be towed by a tracked vehicle. Two methods of towing are provided, the full trail and the semitrail. In the full trail position all three wheels of the gun carriage ride on the ground and towing is accomplished by means of a drawbar. In the semitrail position the dolly or front wheel of the gun carriage rests on the rear platform of the towing vehicle and only the two rear wheels of the gun carriage ride on the ground. Since the semitrail position is used at higher velocities, it will produce the greatest accelerations and consequently this method of towing was investigated.

GENERAL CHARACTERISTICS OF THE SUSPENSION SYSTEM

Some general characteristics of the suspension system may be obtained from an examination of the equation of motion of the system. The equation of motion for the Vigilante gun carriage towed in the semitrail position is developed in Appendix A and shown below.

$$
\begin{aligned}
&\Sigma M_i\ L_i g \cos\beta + M_1 g\left[L_a \cos f_1\ (\beta, S_o t) + L_c \cos\beta\right] \\
&+ M_1\ L_a\left[\ \frac{d^2 f_2}{dt^2}\,(\beta, S_o t)\ \sin f_1\ (\beta, S_o t)\ -\ \frac{d^2 f_3}{dt^2}\,(\beta, S_o t)\ \cos f_1\ (\beta, S_o t)\right. \\
&\left. -\ g \cos f_1\ (\beta, S_o t)\right] - K\left[(\beta - \beta_o + \eta_o - f_1\ (\beta, S_o t)\right] \\
&\times\left[\frac{L_a\ \cos\{f_4\ (\beta, S_o t)\ -\ f_1\ (\beta, S_o t)\}\ +\ L_c\ \cos\ \{f_4(\beta, S_o t)\ -\ \beta\}}{L_a\ \cos\ \{f_4\ (\beta, S_o t)\ -\ f_1\ (\beta, S_o t)\}}\right. - \Sigma M_i\ L_i^{\,2}\ddot{\beta} \qquad (1) \\
&- M_1\left[\frac{d^2 f_2}{dt^2}\,(\beta, S_o t)\ \{L_a \sin f_1\ (\beta, S_o t)\ +\ L_c \sin\beta\}\right. \\
&\left. +\ \frac{d^2 f_3}{dt^2}\,(\beta, S_o t)\ \{L_a \cos f_1\ (\beta, S_o t)\ +\ L_c \cos\beta\}\ \right] = 0
\end{aligned}
$$

The solution of Equation 1 is of the form

$$\beta = g_1(S_o t). \tag{2}$$

The force, F_n, applied at the wheel, is a function of β and $S_o t$ according to Equation 6A of Appendix A and can be expressed in the following form:

$$F_n = f\ (\beta, S_o t). \tag{3}$$

Since β is a function of $S_o t$ from Equation 2, the force F_n can be expressed as a function of $S_o t$ only:

$$F_n = g_2\ (S_o t). \tag{4}$$

The force F_n is not restricted by the equation of motion and can assume positive or negative values. When the force F_n assumes negative values, according to Equation 1, the equation of constraint will not hold and Equation 1 will become invalid since the ground cannot apply a negative, downward force. Under these circumstances the wheel will fail to follow the ground profile and jump will occur. The lowest velocity, S_o, at which jump occurs will be determined experimentally.

The effectiveness of any suspension system is determined by the amplification factor at the operating speed. If the operating speed is below the resonant speed, no reduction of the applied force or acceleration is possible. Also, if the operating speed is well below the resonant speed for a certain range of spring constant values, a change in the spring constant value within that range may produce negligible change in the response of the system. In this investigation, a number of values of spring constants will be tested to determine their effect.

TEST SETUP

A quarter-scale model of the Vigilante gun carriage, Figure 1, was constructed to study the dynamic behavior of the suspension system. Design of the model is discussed in the section entitled "Model Analysis". The rear wheel of the model was supported by a cam with a sine wave profile, Figure 2. This cam was used to simulate uneven cross-country terrain.

The minimum permissible curvature of the cam was determined by the radius of the model wheel. If rolling without impact was to be avoided, the minimum radius of curvature of the cam must equal the radius of the model wheel. This condition would correspond to extremely severe cross-country travel for the prototype. Since this represented a limiting condition, and a more representative condition was desired, the minimum radius of curvature was arbitrarily set at approximately twice the radius of curvature of the model wheel. The amplitude, A_o, and the wave length λ of the cam profile chosen were $A_o = 1$ inch and $\lambda = 26.7$ inches.

Angular displacement of the model, see Figure 2, occurs as a result of horizontal displacement of the model or the cam. The magnitude of the angular displacement is proportional to the relative horizontal displacement between model and cam. In this instance it was found more convenient to displace the cam and restrain the model in the horizontal direction. The model was thus restrained from horizontal displacement by a pin joint at point 3, Figure 2. The velocity of the cam under these circumstances corresponded to the velocity of the model moving over a fixed ground profile.

The cam was formed about a ring wheel as shown in Figure 3. The cam and ring wheel were supported by a frame and driven by a variable speed motor as shown in the same figure.

INSTRUMENTATION

An accelerometer of the resistance bridge type was mounted on the model as shown in Figure 2. The amplified output signal of the accelerometer was used to drive the horizontal plates of an oscilloscope. Signal calibration was obtained by applying a known acceleration of one g to the accelerometer. The vertical amplifier of the scope was not used. Instead, the signal was recorded on film by means of a continuous motion camera. In this arrangement the film was advanced at a constant speed of 5.07 ips past the lens, and the film length was used as the time axis. The film speed was calibrated before and after each test.

MODEL ANALYSIS

The relations between corresponding model and prototype quantities can be derived from their dimensions provided that the ratio between three corresponding model and prototype quantities with independent dimensions are

known. Three quantities with independent dimensions frequently used are mass ($lb\text{-}sec^2/in$), length (in), and time (sec). The relations beween the three quantities chosen are completely arbitrary, but once established, the ratio between all other corresponding model and prototype quantities is fixed. In this investigation, the ratio between mass, acceleration (ips^2), and time for model and prototype, was chosen arbitrarily as indicated below. The subscripts m and p denote model and prototype quantities, respectively, and k represents the scale factor.

$$M_m = \frac{1}{k^3} M_p$$

$$g_m = g_p$$

$$t_m = \frac{1}{\sqrt{k}} t_p$$

In a true model, all quantities are reduced to the proper scale. A simpler model, called an adequate model, can be obtained when only certain quantities are of interest. In an adequate model, only those quantities which affect the magnitude of the desired quantities must be reduced to scale. In this investigation the acceleration of the undercarriage was to be determined and for this purpose an adequate model was considered satisfactory. These model quantities which affected the magnitude of the acceleration were determined from the equation of motion for the Vigilante gun carriage, Equation 11A derived in Appendix A. The relations between these quantities are given in Table I. Also given in Table I are the actual values of model and prototype quantities used in the design of the model.

Since the width dimension did not appear in the equation of motion, Equation 11A, it was reduced to a more convenient scale in the model. Also, in the derivation of Equation 11A, the rear wheels of the gun carriage were assumed to experience equal displacements. Since these displacements were assumed equal, only one wheel was necessary in the model.

PREDICTION EQUATION

Solution of the equation of motion, Equation 11A, for the acceleration $L_i \ddot{\beta}$, or more particularly, $L_o\ddot{\beta}$, can be obtained from a model for those particular values of the parameters $\Sigma M_i L_i^2$, L_i, S_o, λ, A_o and K tested. Extension of the test results to other values of the parameters, however, is quite difficult by use of Equation 11A. To facilitate extension of test results to other values of the parameters K and S_o/λ, a prediction equation will be developed.

The acceleration $L_o\ddot{\beta}$ can be expressed as a function of those variables and parameters which affect its magnitude as follows:

TABLE I

MODEL AND PROTOTYPE QUANTITIES

Quantity	Scale Factor	Model	Prototype
Mass	$M_m/M_p = 1/k^3$		
Weight	$W_m/W_p = 1/k^3$	232.14 lb	14,875 lb*
Length	$L_m/L_p = 1/k$	$L_{c_m} = 52.7$ in.	$L_{c_p} = 210\text{-}3/4$ in.
		$L_{o_m} = 33$ in.	$L_{o_p} = 132$ in.
Inertia	$I_m/I_p = 1/k^5$	820.3 lb-in-sec^2	830,965 lb-in-sec^2
Gravity	$g_m/g_p = 1$	386.4 ips^2	386.4 ips^2
Wave Length	$\lambda_m/\lambda_p = 1/k$	26.7 in.	106.8 in.
Wave Amplitude	$A_m/A_p = 1/k$	1 in.	4 in.
Spring Constant	$K_m/K_p = 1/k^4$	See Table II	
Time	$t_m/t_p = 1/\sqrt{k}$		
Linear Acceleration	$L_o\ddot{\beta}_m/L_o\ddot{\beta}_p = 1$	See Table III	
Linear Velocity	$L_o\ddot{\beta}_m/L_o\ddot{\beta}_p = 1/\sqrt{k}$	See Table III	
Angular Acceleration	$\ddot{\beta}_m/\ddot{\beta}_p = k$		
Angular Velocity	$\ddot{\beta}_m/\ddot{\beta}_p = \sqrt{k}$		
Angle	$\beta_m/\beta_p = 1$		

*Excluding dolly weight

$$1 = C_\alpha\left(\Sigma M_i L_i^2\right)^{C_a}\left(K\right)^{C_b}\left(S_o\right)^{C_c}\left(\lambda\right)^{C_d}\left(L_o\right)^{C_e}\left(A_o\right)^{C_f}\left(t\right)^{C_g}\left(L_o\ddot{\beta}\right)^{C_h} . \qquad (5)$$

By use of dimensional analysis, Equation 5 can be reduced to the following form:

$$1 = C_\alpha\left(\frac{A_o}{L_o}\right)^{C_1}\left(\frac{A_o}{\lambda}\right)^{C_2}\left(\frac{S_o t}{\lambda}\right)^{C_3}\left(\frac{S_o^2 L_o}{\lambda^2 L_o\ddot{\beta}}\right)^{C_4}\left(\frac{L_o\ddot{\beta}\Sigma M_i L_i^2}{KL_o}\right)^{C_5} . \qquad (6)$$

Since the design of the prototype has been established, no variation of the parameters M_i or L_i is contemplated and these parameters may be treated as constants and included in the coefficient term. The acceleration $L_o\ddot{\beta}$ can then be expressed in terms of the remaining variables and parameters as follows:

$$L_o\ddot{\beta} = C_\alpha' \; A_o^{C_1'}\left(\frac{A_o}{\lambda}\right)^{C_2'}\left(\frac{S_o t}{\lambda}\right)^{C_3'}\left(\frac{S_o^2}{\lambda 2}\right)^{C_4}\left(\frac{1}{K}\right)^{C_5} . \qquad (7)$$

The term $\left(\frac{S_o t}{\lambda}\right)^{C_3'}$ in Equation 7 above can be expanded in series form as follows:

$$\left(\frac{S_o t}{\lambda}\right)^{C_2'} = \sum_{j=1}^{j=n}\left(A_j \sin\frac{j\pi}{\lambda} S_o t + B_j \cos\frac{j\pi}{\lambda} S_o t\right). \qquad (8)$$

A Fourier Series expansion for $\left(\frac{S_o t}{\lambda}\right)^{C_2'}$ was chosen in this instance since the solutions for $L_o\ddot{\beta}$, Figure 4, obtained from the model indicate that the acceleration $L_o\ddot{\beta}$ is periodic and thus $\left(\frac{S_o t}{\lambda}\right)^{C_3'}$ must be periodic. Further examination of the solutions for $L_o\ddot{\beta}$ indicate that it has a fundamental frequency of $\frac{S_o}{\lambda}$ and contains certain other higher frequencies associated with the natural frequency of the suspension system. Since the higher order frequencies have little effect on the magnitude or the acceleration $L_o\ddot{\beta}$, it can be expressed in terms of the fundamental frequency with little resultant error. Thus

$$L_o\ddot{\beta} = C_\alpha'\left(A_o\right)^{C_1'}\left(\frac{A_o}{\lambda}\right)^{C_2'}\left(\frac{S_o^2}{\lambda^2}\right)^{C_4'}\left(\frac{1}{K}\right)^{C_5'} \left[A_1 \sin\frac{\pi}{\lambda} S_o t + \beta_1 \cos\frac{\pi}{\lambda} S_o t\right] . \qquad (9)$$

In this investigation only the maximum acceleration is of interest and this may be expressed as follows:

$$L_o\ddot{\beta}_{max} = C_\alpha''\left(A_o\right)^{C_1'}\left(\frac{A_o}{\lambda}\right)^{C_2'}\left(\frac{S_o^2}{2}\right)^{C_3'}\left(\frac{1}{K}\right)^{C_4'} \tag{10}$$

since $[A_1 \sin\frac{\pi}{\lambda}S_o t + \beta \cos\frac{\pi}{\lambda} S_o t] = \text{Constant.}$

The unknown exponents of Equation 10, C_1', C_2', C_3', and C_4', together with the coefficient C_α'' must be evaluated from the test results. In this investigation, however, only one value of A_o and λ were used and therefore the exponents C_1' and C_2' were not determined. This does not constitute a serious limitation, however, since the acceleration produced by different spring constants and different speeds can still be compared at the same value of A_o and λ. The ratio of these accelerations at different values of A_o and λ does not change and thus the optimum spring constant can still be determined. For constant values of A_o and λ, the acceleration can be expressed as follows:

$$L_o\ddot{\beta}_{max} = C_\alpha''' \frac{S_o^{2\,C_3'}}{K^{C_4'}} \tag{11}$$

The value of the exponents C_3' and C_4' and the value of the coefficient C_α''' were determined from the test results as follows:

$$\frac{(L_o\ddot{\beta}_{max})S_{o1}}{(L_o\ddot{\beta}_{max})S_{o2}} = \left(\frac{S_{o1}^2}{S_{o2}^2}\right)^{C_3'} \qquad K = \text{constant} \tag{12}$$

$$\frac{(L_o\ddot{\beta}_{max})K_1}{(L_o\ddot{\beta}_{max})K_2} = \frac{(S_o^2)K_1}{(S_o^2)K_2}\left(\frac{K_2}{K_1}\right)^{C_4'} \tag{13}$$

where K_1 and K_2 represent particular values of K and

$$C_\alpha''' = \frac{L_o\ddot{\beta}_{max}}{\dfrac{S_o^{2C_3'}}{K^{C_4'}}} \tag{14}$$

The final form of the prediction equation, including the values for C_α''', C_3', and C_4 determined as outlined above was

$$L_o\ddot{\beta}_{max} = .19 \frac{S_o^2}{K^{.18}} \text{ ips}^2 . \qquad (15)$$

It should be emphasized that Equation 15 is strictly valid only for a ground profile in the form of a sine wave and for the particular values of amplitude and wave length tested.

TESTS AND TEST PROCEDURE

A series of four tests were conducted with the model using different value of spring constant for each test. The value of the spring constants used are shown in Table II. In each test the speed of the cam was increased until jump occurred. The acceleration $L_o\ddot{\beta}$ as measured by the accelerometer was recorded on film. Traces of the acceleration signal are shown in Figure 4.

TABLE II

Test No.	K (in-lb/rad)
1	3,288
2	4,790
3	10,000
4	36,000

COMPUTATIONS AND TEST RESULTS

The accelerations $L_o\ddot{\beta}$ were read directly from the film traces. In all traces 5 lines represent a one g acceleration. The velocity of the cam S_o was determined as follows:

$$f = \frac{S_o}{\lambda} = \frac{v}{L_f}$$

where f = frequency of oscillation, cyc/sec

S_o = speed of cam, ips

λ = wave length of ground profile, in.

v = velocity of film, ips

L_f = wave length of trace on film, in.

The test results are shown in Table III.

TABLE III

MODEL TEST RESULTS

K (in-lb/rad)	$L_o\ddot{\beta}_{max}$ (g)	$L_o\ddot{\beta}_{min}$ (g)	L_f (in.)	S_o (mph)	Comments
3,290	+ .8 + .95 +1.0	-.7 -.85 -.85	1.61 1.48 1.43	4.78 5.20 5.38	Jump Velocity
4,790	+ .95	-.85	1.41	5.4	Jump Velocity
10,000	+ .95 +1.0	-.75 -.85	1.46 1.31	5.28 5.86	Jump Velocity
35,000	+ .95	-.85	1.2	6.42	Jump Velocity

λ = 26.7 in., A_o = 1 in.

CONCLUSIONS

1. The maximum acceleration, $L_o\ddot{\beta}_{max}$, is approximately one g for the range of spring constant values tested and increases slightly as the spring constant K is increased.

2. The velocity at which jump occurs is approximately 5 mph for the conditions tested and increases slightly as K is increased.

3. The optimum value of K as determined in this investigation is the largest value of K tested. This is based on the fact that the maximum accelerations $L_o\ddot{\beta}_{max}$ in all cases tested are well below the maximum permissible acceleration and that the largest value of K produced the highest velocity of jump.

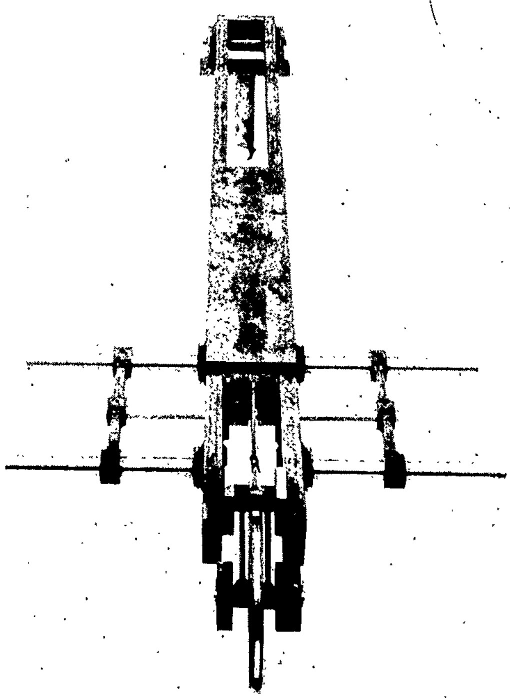


Figure 1. VIGILANTE GUN CARRIAGE

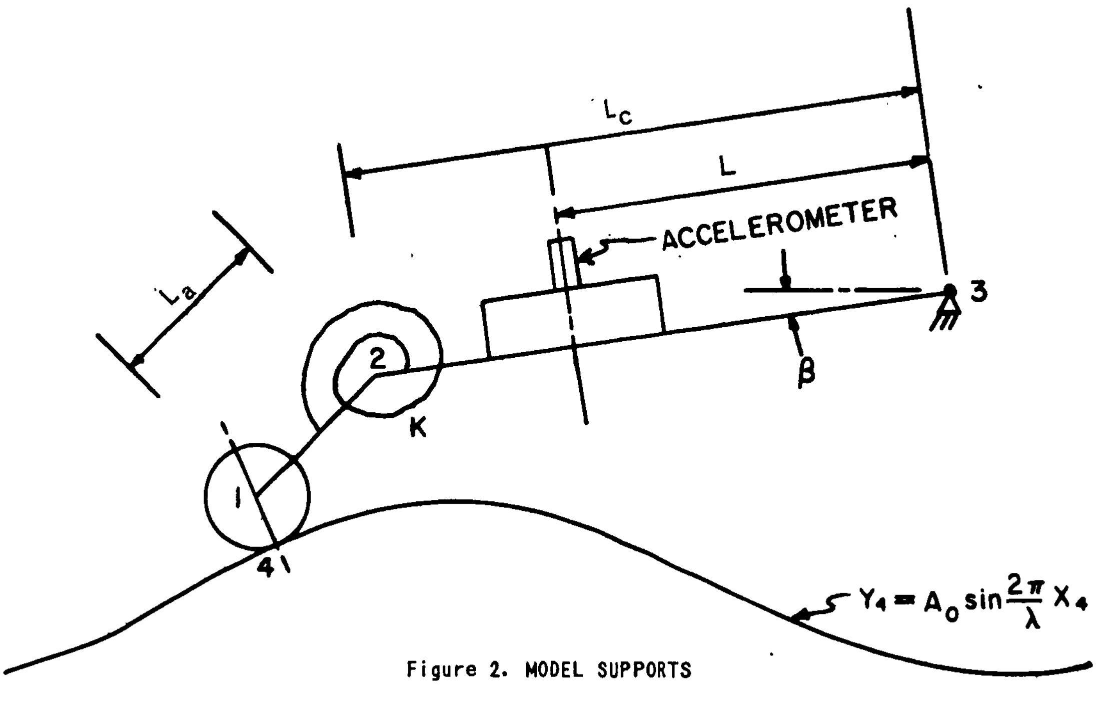


Figure 2. MODEL SUPPORTS

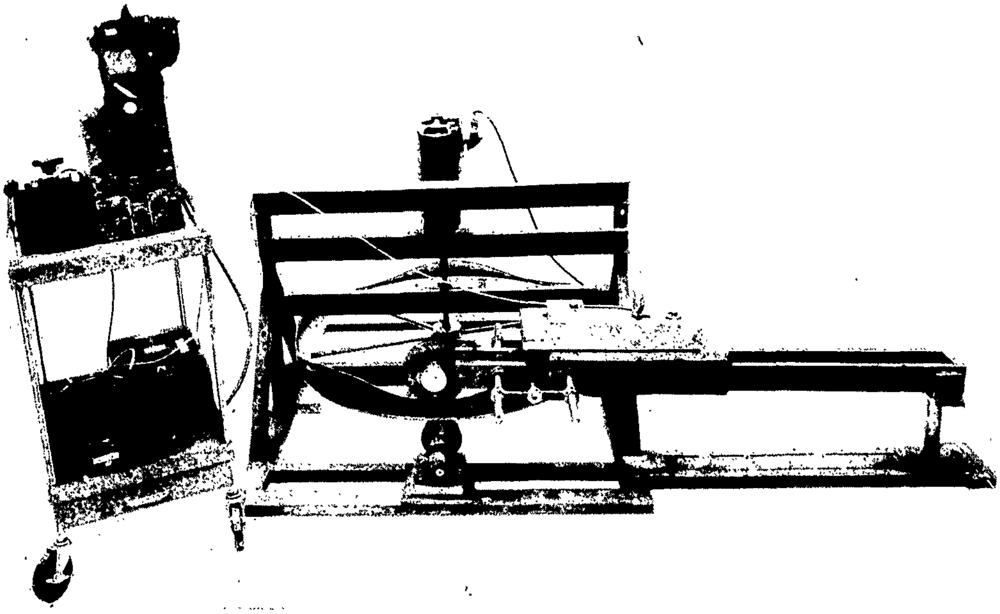


Figure 3. TEST SETUP

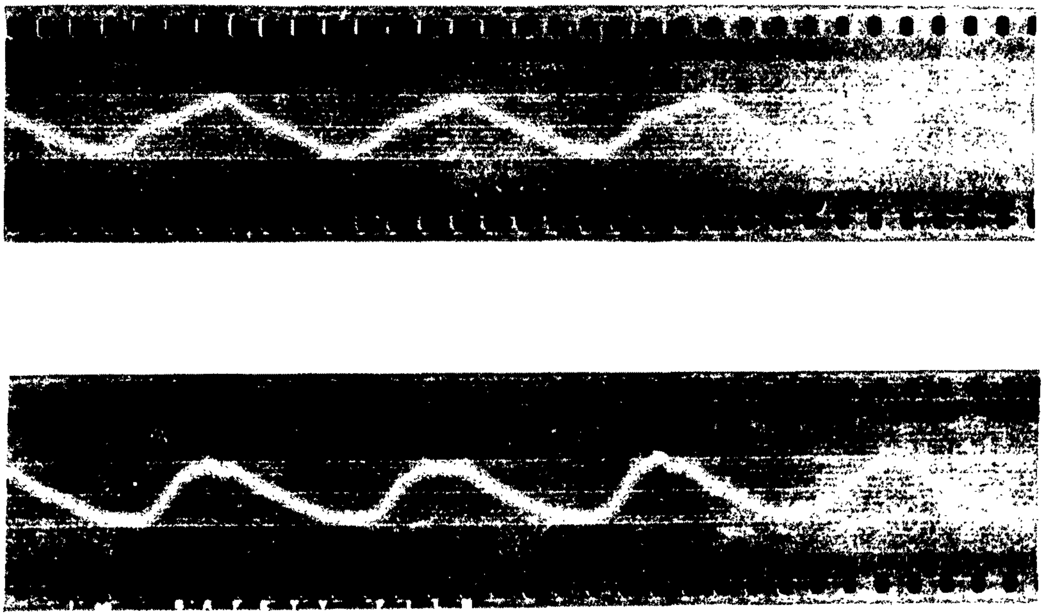


Figure 4. ACCELERATION TRACE

APPENDIX A

The equation of motion for the Vigilante gun carriage, towed in the semitrail position, will be derived by use of D'Alembert's Principle.

The gun carriage is shown in the semitrail position in Figure A-1. When towed in this manner, the front wheel of the gun carriage rests on the rear of the towing vehicle. The dynamics of the towing vehicle are thus associated with the dynamics of the Vigilante gun carriage. In this investigation, however, the characteristics of the Vigilante suspension system are of primary importance and these characteristics are independent of the type of forward support. The rear suspension system of the Vigilante gun carriage will thus be analyzed independently by assuming that point 3 of Figure A-1 has constant horizontal velocity and no vertical motion. With these assumptions, the equation of motion of the Vigilante gun carriage can be written by summing the torques about point 3. See Figure A-1.

$$\Sigma T_3 - \Sigma M_i \, L_i^2 \ddot{\beta} - M_1 \, [\ddot{x}_1 \, (L_a \sin \eta + L_c \sin \beta) - \ddot{y}_1 \, (L_a \cos \eta + L_c \cos \beta)] = 0 \tag{1A}$$

where

$$\Sigma T_3 = \Sigma M_i \, L_i g \cos \beta + M_1 g \, (L_a \cos \eta + L_c \cos \beta)$$

$$- F_n [\sin \psi \, (L_a \sin \eta + L_c \sin \beta) + \cos \psi \, (L_a \cos \eta + L_c \cos \beta)] \tag{2A}$$

Equation 2A may be simplified by use of trigonometric identities as follows:

$$\Sigma T_3 = \Sigma M_i \, L_i g \cos \beta + M_1 g \, (L_a \cos \eta + L_c \cos \beta)$$

$$- F_n \, [L_a \cos (\psi - \eta) + L_c \cos (\psi - \beta)] \tag{3A}$$

where

$$[\sin \psi \, (L_a \sin \eta + L_c \sin \beta) + \cos \psi \, (L_a \cos \eta + L_c \cos \beta)] = L_a \cos (\psi - \eta) + L_c \cos (\psi - \beta)$$

The normal force F can be expressed as a function of the spring constant K and the angles η and β by summing the torques about point 2 and solving for F_n.

$$\Sigma T_2 - M_1 L_a (\ddot{x}_1 \sin \eta - \ddot{y}_1 \cos \eta) = 0 \qquad (4A)$$

where

$$\Sigma T_2 = M_1 g L_a \cos \eta - F_n L_a (\cos \psi \cos \eta + \sin \psi \sin \eta) - K (\beta - \beta_o + \eta_o - \eta) \qquad (5A)$$

thus

$$F_n = \frac{-M_1 L_a (\ddot{x}_1 \sin \eta - \ddot{y}_1 \cos \eta - g \cos \eta) - K (\beta - \beta_o + \eta_o - \eta)}{L_a (\cos \psi - \eta)} \qquad (6A)$$

Substitution of Equations 2A and 6A into Equation 1A yields

$$\Sigma M_i L_i g \cos \beta + M_1 g (L_a \cos \eta + L_c \cos \beta) +$$

$$\frac{M_1 L_a (\ddot{x}_1 \sin \eta - \ddot{y}_1 \cos \eta - g \cos \eta) - K (\beta - \underline{\beta_o} + \eta_o - \eta)}{L_a (\cos \psi - \eta)} [L_a \cos (\psi - \eta) + L_c \cos (\psi - \beta)]$$

$$- \Sigma M_i L_i^2 \ddot{\beta} - M_1 [\ddot{x}_1 (L_a \sin \eta + L_c \sin \beta) + \ddot{y}_1 (L_a \cos \eta + L_c \cos \beta)] = 0 \qquad (7A)$$

The displacements x_1 and y_1 and the angle η are functions of β and $S_o t$ Equations 4B to 6B, Appendix B, when the conditions of constraint are maintained.

Thus

$$\ddot{x}_1 = \frac{d^2}{dt^2} f_2 (\beta, S_o t) \qquad (8A)$$

$$\ddot{y}_1 = \frac{d^2}{dt^2} f_3 (\beta, S_o t) \qquad (9A)$$

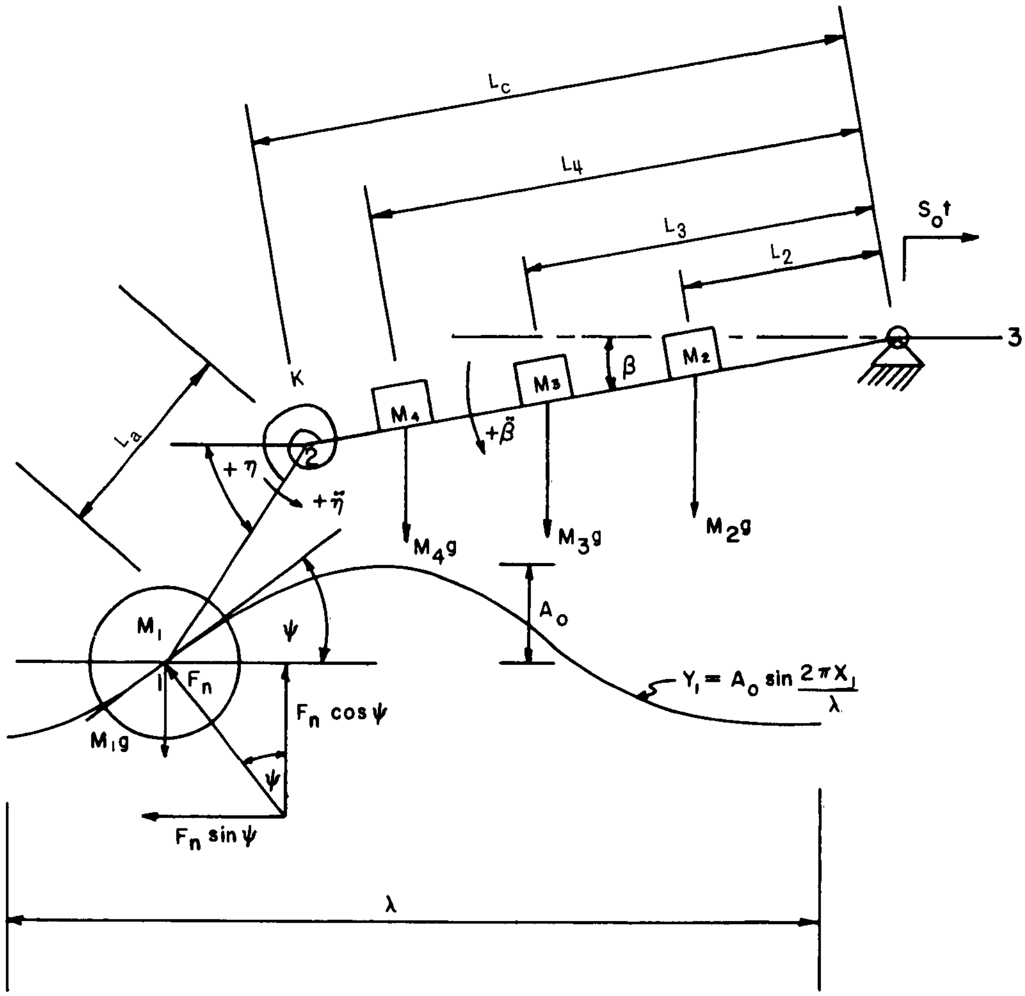
The angle ψ is formed by the tangent to the ground profile at the point x_1 and the horizontal plane, thus,

$$\tan \psi = \frac{dy_1}{dx_1}$$

$$\psi = \tan^{-1} \frac{dy_1}{dx_1} = k (\beta, S_o t) \qquad (10A)$$

When Equations 8A and 9A above and Equations 4B to 6B, Appendix B, are substituted into Equation 7A above, the equation of motion takes the following form:

$$
\begin{aligned}
&\Sigma M_i\, L_i g \cos\beta + M_1 g \{L_a \cos f_1\, (\beta, S_o t) + L_c \cos\beta\} \qquad\qquad (11A)\\
&+ \left\{ M_1\, L_a \left[\left\{ \frac{d^2 f_2}{dt^2}\, (\beta,\ S_o t) \sin f_1\, (\beta,\ S_o t) \right\} - \frac{d^2 f_3}{dt^2}\, (\beta,\ S_o t) \cos f_1\, (\beta,\ S_o t) \right.\right.\\
&\left.\left. - g \cos f_1\, (\beta,\ S_o t) \right] - K\, (\beta - \beta_o + \eta_o - f_1\, (\beta,\ S_o t) \right\} x\\
&\frac{\left[L_a \cos \{f_4\, (\beta,\ S_o t) - f_1\, (\beta,\ S_o t)\} + L_c \cos \{f_4\, (\beta,\ S_o t) - \beta\}\right]}{L_a \cos \{f_4\, (\beta,\ S_o t) - f_1\, (\beta,\ S_o t)\}} - \Sigma M_i L_i^2\, \ddot{\beta}\\
&- M_1 \left[\frac{d^2 f_2}{dt^2}\, (\beta,\ S_o t)\, \{L_a \sin f_1\, (\beta,\ S_o t) + L_c \sin\beta\} + \frac{d^2 f_3}{dt^2}\, (\beta,\ S_o t)\ x \right.\\
&\left. \{L_a \cos f_1\, (\beta,\ S_o t) + L_c \cos\beta\} \right] = 0
\end{aligned}
$$



FREE BODY DIAGRAM

FIGURE A-1

APPENDIX B

The Vigilante gun carriage as shown in Figure A-1 has three degrees of freedom η, β and S_0t. If no constraint is imposed on the wheel, it will, under certain operating conditions, lose contact with the ground profile and jump with subsequent impact will result. Under different, less severe operating conditions, the wheel will maintain contact with the ground profile and for these conditions certain relations exist between the variables η, β, and S_0t. In the theoretical analysis presented here, it will be assumed that the less severe operating conditions apply and that the wheel maintains contact with the ground profile. It will also be assumed that the center of the wheel, point 1, Figure A-1, has the same displacement as the actual point of contact, point 4, Figure 2.

The relations between the variables η, β, and S_0t for the condition of constraint assumed above may be developed from the relations shown in Figure A-1.

$$L_a (\cos \eta_0 - \cos \eta) + L_c (\cos \beta_0 - \cos \beta) = x_1 - S_0 t \qquad (1B)$$

$$L_a (\sin \eta_0 - \sin \eta) + L_c (\sin \beta_0 - \sin \beta) = y_1 \qquad (2B)$$

$$y_1 = A_0 \sin \frac{2\pi}{\lambda} x_1 \qquad (3B)$$

Equations 1B to 3B above contain five variables x_1, y_1, η, β, and S_0t, thus only two of the variables are independent and the remaining three variables are functions of the two independent variables. If β and S_0t are chosen as the independent variables, the remaining three variables may be expressed as follows:

$$\eta = f_1 (\beta, S_0 t) \qquad (4B)$$

$$x_1 = f_2 (\beta, S_0 t) \qquad (5B)$$

$$y_1 = f_3 (\beta, S_0 t) \qquad (6B)$$

The above equations 4B to 6B are the equations of constraint for the conditions specified.

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